The nonlinear GDP dynamics[5]

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Abstract

The oscillatory behavior of GDP and its components leads to a Fourier transform analysis that results in the eigen values of the dynamic economic system. The larger values are dominating the transient behavior of the GDP components and these transients are discussed along with the specific behavior of each component. The second order differential equations are determined, for each component, to describe the oscillatory behavior and the transient resulting from a step excitation. The natural frequencies are determined and the correlation of pairs of components' cycles result in 'beats' process where the modulated wave cycles are compared and discussed. Based on these correlated influence terms the solutions of the differential equations of each component are determined along with their evolution in the phase space and with the specific Lagrangian. The possible occurrence of dynamical behavior basins for GDP is explored, associated to time interval least action considerations. Important conclusions are drawn from this analysis on the dynamics of GDP and its components, in terms of general versus local equilibriums and their evolution.

Keywords: nonlinear models, oscillatory behavior, GDP cycles, equilibrium

JEL Classification: C3, C61, C62, D7, D87

1. Introduction

Oscillatory processes are imbedded in economic systems' behavior due to the intrinsic nature of human activities and natural phenomena with which they interact. Various names are coining this behavior such as cyclic, seasonal, yearly or quarterly periodic, etc. Moreover the scale of time constants of various activities range from seconds e.g. the ticks of the stock exchange, to tens of years e.g. T bonds of the US Treasury.

As shown in the selected references, there are various papers that have identified nonlinear behavior and described it with various models that show features such as bifurcation, discontinuity, periodicity, etc. What we intend to present here is a systematic analysis of the oscillatory behavior of the GDP and its components that applies well known mathematical instruments e.g. Fourier transform, differential equations, flows, Lagrangean etc. We give below, in chronological order, a few relevant quotes related to cyclic behavior:

"The general character and agreement in the periodic turn in movements of factors of circulation -- these are the specific problems of business cycle theory which have to be solved within the closed interdependent system. … If a business cycle theory which is *system-*

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conforming cannot be built, then "general overproduction" will not only drive the economy but also economic *theory* into a crisis." (Adolph Lowe, "How is Business Cyle Theory Possible at All?", *WWA*, 1926: p.175).

"Since we claim to have shown in the preceding chapters what determines the volume of employment at any time, it follows, if we are right, that our theory must be capable of explaining the phenomena of the Trade Cycle." (John Maynard Keynes, *General Theory*, 1936: p.313)

"Keynesian economics, in spite of all that it has done for our understanding of business fluctuations, has beyond all doubt left at least one major thing quite unexplained; and that thing is nothing less than the business cycle itself....For Keynes did not show us, and did not attempt to show us, save by a few hints, why it is that in the past the level of activity has fluctuated according to so definite a pattern." (John Hicks, *Contribution to the Theory of the Trade Cycle*, 1950: p.1)

Further on, one should mention Samuelson, Minsky and Gandolfo, as well as S.Keen, that contributed substantially to the formulation of models that present oscillatory dynamics and to the discussion of their various basins of behavior. As an example, Keen says;

“The economic fixation upon equilibrium appears quaint to these mathematically literate economists, and this alone may significantly undermine the hold which static thinking has on economics” (Steve Keen, *Debunking Economics*, 2007: p309)

We will leave for the references the multiple papers at the basis of our survey of models imbedding cyclic behavior done by Hicks, Dussenbery, Kalecki, Kaldor, Goodwin, etc. that either introduce limitation such as ceilings and floors or consider nonlinear components that allow the occurrence of cycles.

2. Complex systems behavior

2.1. Economies as complex systems

Complex systems theory has developed specific concepts that may help to focus the description of economic behavior Following Kay and Regier (2000), one may identify some characteristics of complex systems, as dealt with later, as follows:

- non-linear behavior (generated also by feedback);
- hierarchical structure (layers of nested systems made up of systems);
- internal causality (self-organizing causality characterized by objectives, positive and negative feedback, autocatalysis, emergent properties, and sudden change);
- the fact that there may not exist single equilibrium points; and that multiple attractor points (steady states) are possible;
- they show catastrophic behavior, with bifurcations and flips between attractors; and even chaotic behavior, where our ability to forecast and predict is limited.

Economies are observing the above characteristics of complex adaptive systems, i.e. composed of large and increasing number of both components and of the relationships between them.

They also learn from mistakes and from present developments, and they react, by changing both the actions undertaken and the objectives defined; they are thus self-reflexive.

Economies are also systems having an objective, or goal, they tend to self-maintenance, development and they are capable of incorporating the deduced consequences of achieving their objectives into the present decisions and definitions of new objectives, they are thus anticipatory.

They also have the ability to adapt to new changing limits (seen as boundary conditions), but they may consciously alter the limits (e.g. in their relation with the environment). This is why the economy, as a human system, can be understood as a complex, adaptive, self-reflexive, and self-aware system (see Kay and Regier, 2000 and Purica and Martinelli 1992).

The study of the evolution under the complex system framework will help us to better conceptualize the relationship between the development of economies and their equilibrium dynamic evolution. For a deeper analysis on economies as complex systems, see Ramos-Martin (2002).

2.2. GDP components and energy evolution as indicators of a complex system framework

The hierarchical structure of the economy, as well as the working of the feedback loops between the different hierarchical levels, induces non-linear behavior in the system. This is so because positive feedback loops might generate self-reinforcing mechanisms. This non-linear behavior is not only induced by external shocks as is normally implied by economic theory, but also by internal causes within the system. Both non-linear behavior and far from equilibrium situations, lead to the existence of several stable states (Proops, 1985) or attractors.

An attractor represents a region, in the phase space, where the behavior exhibited by the system is coherent and structured (Kay et al., 1999). Once the system reaches the attractor, it fluctuates in that region and its parameters move only short distances, at least for a certain period of time. This is known as 'lock-in', and prevents the system from taking another trajectory for a period of time (Dyke, 1994; Kay et al., 1999).

If we study the evolution of the parameters above and we state that the system is on an "attractor point" some changes in, for example, GDP components, will not always lead to changes in the studied variable.

Over half a century ago, Schumpeter (1949) understood non-linear evolutionary development and discontinuity by means of his theory of creative destruction. This idea has been later named 'punctuated equilibrium' by some analysts (Gowdy, 1994), using the same term that is in use in paleontology to describe this step-wise evolution (Eldredge and Gould, 1972; Gould and Eldredge 1993).

One way of analyzing the existence of this discontinuity is by means of a phase diagram. This methodology has been used in the case of CO2 emissions (Unruh and Moomaw, 1998), and in the case of energy intensity (De Bruyn, 1999; Ramos-Martin 1999, 2001). The phase diagrams are intended to show whether the evolution of certain variables, over time, is stable (even dynamically stable) or divergent. Thus one may find if there are attractor points or not.

The fact that a particular system is evolving around one attractor point constrains the future available trajectories and attractors, by paving the path for future developments on a least action based evolution. The system moves along a trajectory that passes from a given attractor in a given time period to another attractor in the next time period. In order to make a calculable analysis we are going to make specific considerations on the system. In a simple case we may consider that the system (economy) behavior is an oscillatory one where GDP components are each described by oscillations resulting from various causes not to be detailed at this point. To each a second order ordinary differential equation is associated. The evolution of the GDP is actually not a sum of components but a superposition of oscillations. Seeing it in this way the combinations of oscillations are resulting in modulated short and long time periods (i.e. high and low frequencies) waves. This modulated behavior creates a sum of excitation terms that act as an excitation in the differential equations describing each GDP component. Two main things are looked for in the second order differential equations: (i) the pattern of solutions in the phase space that may indicate convergent behavior and thus, the existence of an attractor (indicating stability) or divergence that may trigger the onset of change to an unstable behavior; (ii) the existence of a Lagrangean function of the system such as to identify the types of terms associated to the energy equivalent of the system (e.g. elastic terms, kinetic terms, potential terms).

Once we have determined the Lagrangean functions one may consider analyzing the existence of extremes that would indicate points of equilibrium toward which the system would go in each time period. Since we talk about time periods and the Lagrangean one may think of the functional integral of the Lagrangean (measuring the Action equivalent of the system) to identify the equilibrium points, that the system will path through from one time interval of interest to the next. This least action technique, (path integral), was introduced by Feynman in 1942 (Feynman, 2005). It integrates the time dependent coefficients of a Lagrangean over each time interval of interest in order to identify the points of equilibrium of the system that its evolution trajectory is passing through. Obviously these equilibrium points may not be the same from one time interval to the next.

Further on we will consider the data series of GDP and its components and apply the approach described above. The actual data are from Romania and have a quarterly disaggregation.

3. Case analysis Romania

By applying this type of analysis to the real GDP data from a given economy (Romania) we obtain a set of second order differential equations. Each of these equations has its specific solutions. Actually the resulting amplitudes of the Fourier analysis could be considered eigen-values of these equations and the flow of trajectories may be discussed in terms of convergence/divergence, stability and the underlying potential.

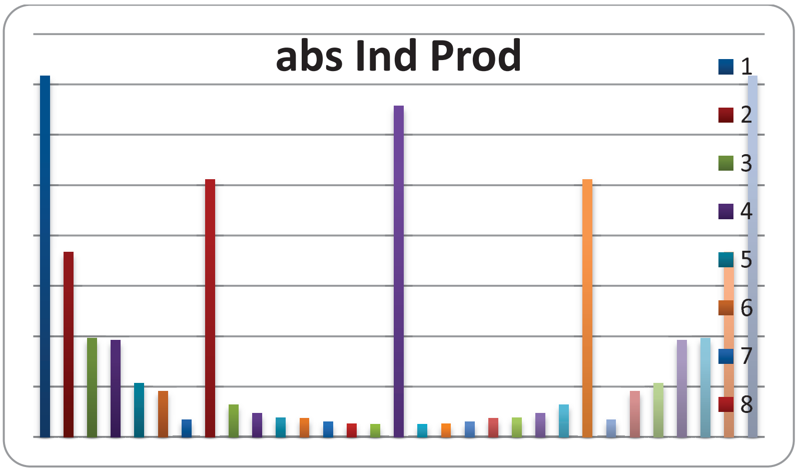
We have done a basic analysis of this sort (Purica 2011) in a previous paper for the industrial production. In what follows we are extending the analysis to the GDP and all its components.

3.1. Data and Fourier analysis

The data of the GDP and its components are taken as quarterly series in the period Q1 2000 – Q4 2008 (before the crisis starting). The source is the National Institute of Statistics of Romania. The values are retrievable from the site of the institute. We have taken the data given in current values, and performed the Fourier analysis on the series, taking the last 32 values of each, as required by the Fourier calculation. Although we have calculated the values for all the GDP components we are keeping here only the industrial production (a convergent component) and finance (a divergent one) for the sake of showing our point. Some results will though consider the full data calculations. The resulting graphs of the amplitudes are given below.

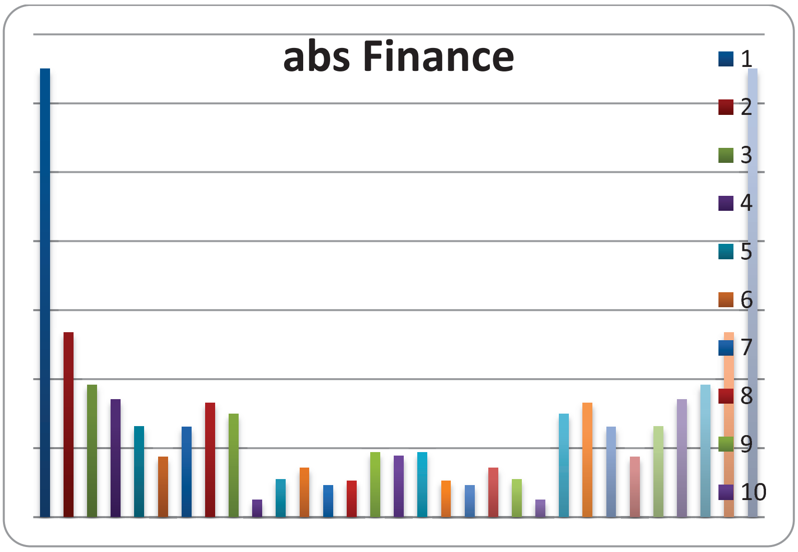
It is important to note that the representations above are only giving the real values of the amplitudes. The representation in the complex plane may actually be considered as the one that gives the eigen values of each component us determining the flow of trajectories.

Industrial Production



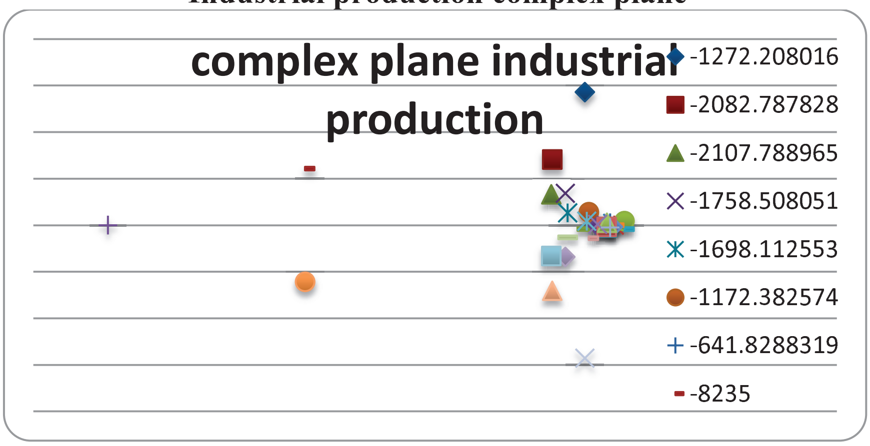
Source: Author's calculations

Finance



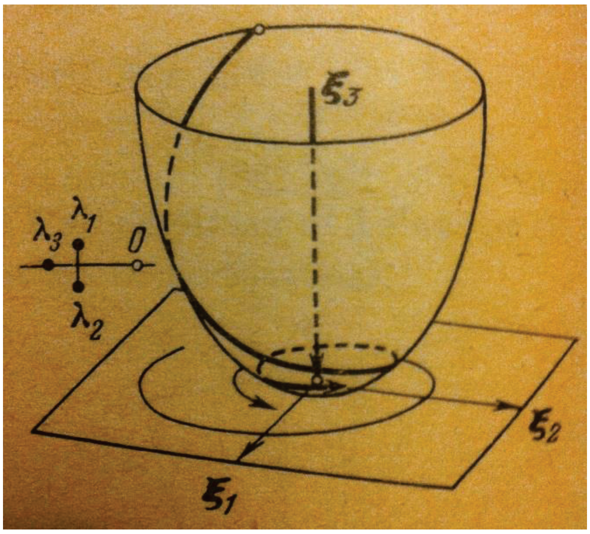
Source: Author's calculations

Industrial production complex plane



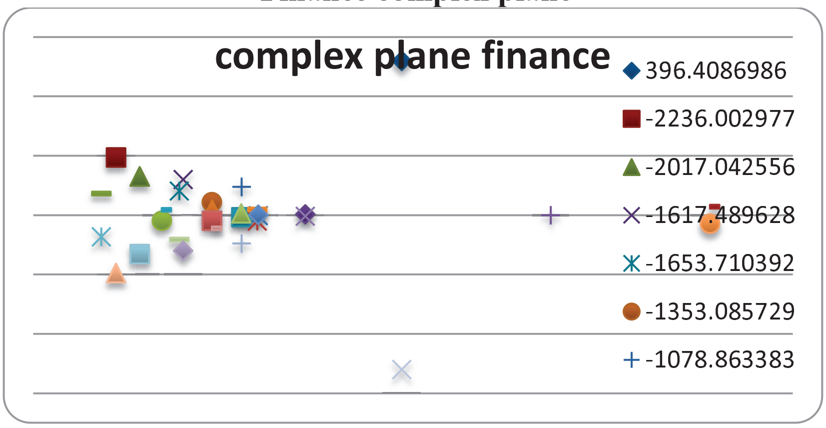
Source: Author's calculations

Industrial Production flow



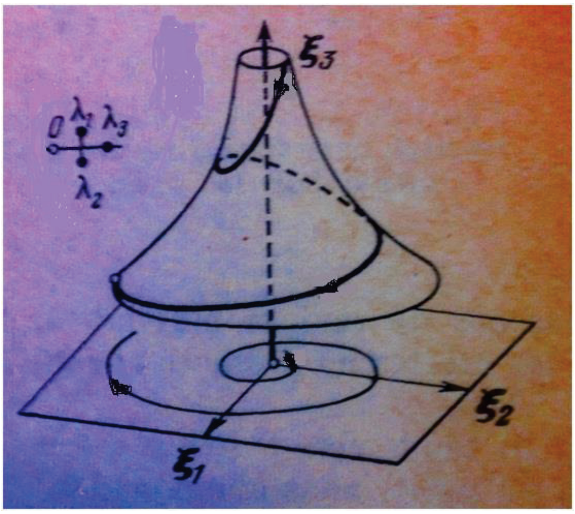
Source: Author's calculations (see also Arnold 1974)

Finance complex plane



Source: Author's calculations

Finance flow



Source: Author's calculations (see also Arnold 1974)

3.2. Determination of the differential equations

The differential equations describing the oscillatory solutions have been determined for each component in two cases: (i) a case of unperturbed evolution that results in the specific frequencies of each case and, (ii) a case of perturbed evolution by a step excitation that gives the natural and transient frequencies, to be used later in the calculations. We are giving here only the perturbed case. The equation for each component is:

$$M\frac{d^2}{d\square^2}x(\square)+2\frac{c}{2}\frac{d}{d\square}x(\square)+kx(\square)=u(\square)$$

Table 1. Coefficients and dynamic parameters for step function excitation.

Component	M	c	k	T	S	C
AGR	1.026	0.864	1.15	5.934	6.468	2.374
COM	5.452	2.765	5.414	6.305	6.52	3.943
CTR	0.985	0.703	2.428	4.002	4.109	2.802
FIN	1.012	0.824	1.102	6.021	6.538	2.458
IND	2.857	1	3.656	5.554	5.622	5.713
SRV	11.072	6.213	15.907	5.242	5.392	3.564

Legend
AGR - Agricultural production
COM - Commerce
CTR - Constructions
FIN - Finance
IND - Industry
SRV - Services

Further on we will start analyzing the correlation of various sectors. It is clear that there is mutual interdependence among the sectors e.g. finance and industry are interdependent as well as finance and every other sector. Moreover, there is interdependence among the nonfinancial sectors such as industry and agriculture or agriculture and commerce, etc.

3.3. Long and short time inter-sectorial cycles

If one combines two cyclic processes the resulting pattern of behavior results in high frequency cycles (given by the sum of the two frequencies) modulated by lower frequency cycles (given by the difference of the two frequencies). In physics the process is called ‘beats’ and its perception is mostly demonstrated in sound waves.

Up till now we have considered only single GDP component oscillatory behavior, with no interconnection among sectors. Let us now try to assess more information from combining two sectors and analyze the resulting behavior.

The resulting wave is a product of the sine of the frequency sum, sin((wf+wi)/2), by the cosine of the frequency difference, cos((wf-wi)/2). The shape of the resulting wave is giving information not only on short term cyclic behavior but also on the modulating cycle that may imply there is an intrinsic long term cyclic behavior which may be associated with what we usually perceive as ‘crises’, or that have been shown to exists by various authors (of which the most known is Kondratiev).

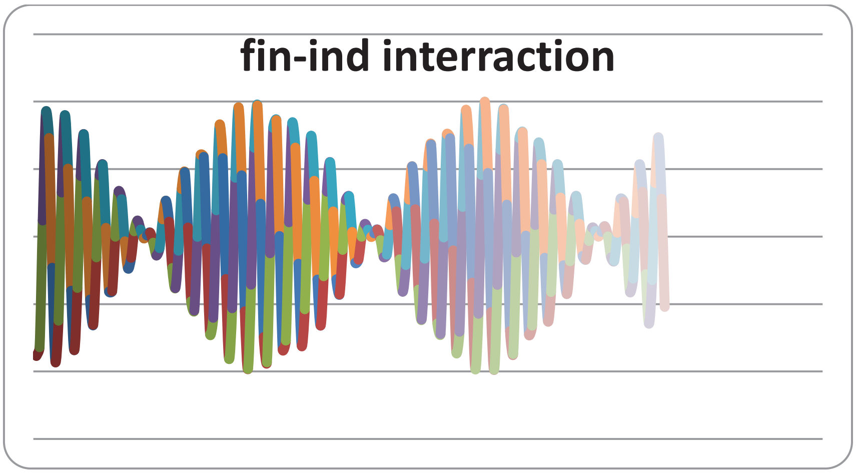
We keep the notations from table 1 above for the sectors (but in small characters) we give the two frequencies for each correlation. Also we note $w=2\pi/T$ (T from table 1 above). As mentioned we are giving only finance and industrial production.

Finance and industry (fin ind): the time units are measured in quarters [Q], so the short period is 2π/(wfin+wind)=2.89 Q i.e. less than one year, while the modulation (long period) is 2π/(wfin-wind)=69.78 Q i.e. about 17 years.

The period of the modulation is of the order of the investment cycles in the industrial sector. We may have found by this method a way to assess the length of the investment cycles in given economies as given by the modulated wave of the correlated financial and industrial cycles of the respective GDP components.

The fin-ind frequencies

wfin-wind	wfin+wind
-0.09	2.17



Source: Author's calculations

The following table is a synthesis of the associated periods with the cycles above. The upper part of the table shows the periods of the modulated cycles calculated in years while the lower part shows the periods of the short cycles given in months.

Table 3. Intersectorial cycles periods long [years] and short [months]

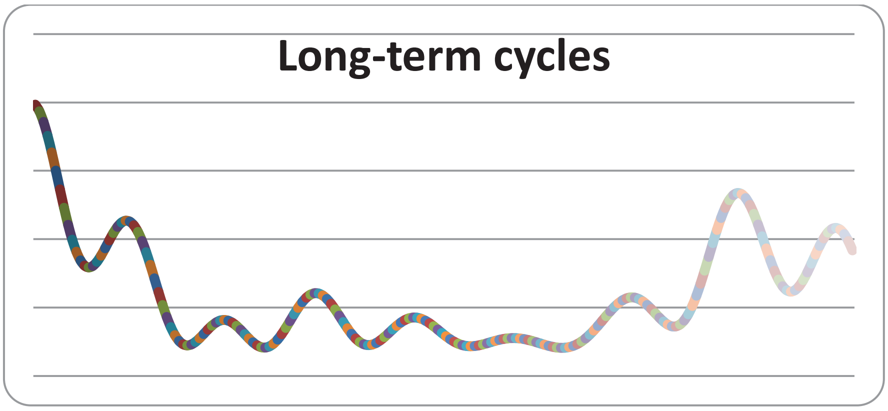
	fin	ind	agr	com	ctr	srv	
fin	1	17	79	31	3	10	years
ind	*9*	1	22	12	4	22	
agr	*9*	*9*	1	26	3	11	
com	*9*	*9*	*9*	1	3	8	
ctr	*7*	*7*	*7*	*7*	1	4	
srv	*8*	*8*	*8*	*9*	*7*	1	
months							

The modulated cycles are the ones with larger time constants that describe behavior that covers, in certain cases, tens of years. These long modulated cycles are the ones that supposedly generate the long-term variations that we normally perceive as crises, being different from the usual quarterly, or at most, yearly behavior.

The cycle's short term and long term periods may become a set of indicators for the expected behavior; for example, construction correlated with finance, services and commerce show similar values periods for short term behavior and also for long term one (also relatively small). This high dynamic of the above correlations leads to caution related to the domain of constructions where a bubble situation may occur. Actually in the years following the interval we have analyzed (2000-2008) this situation has showed up in Romania.

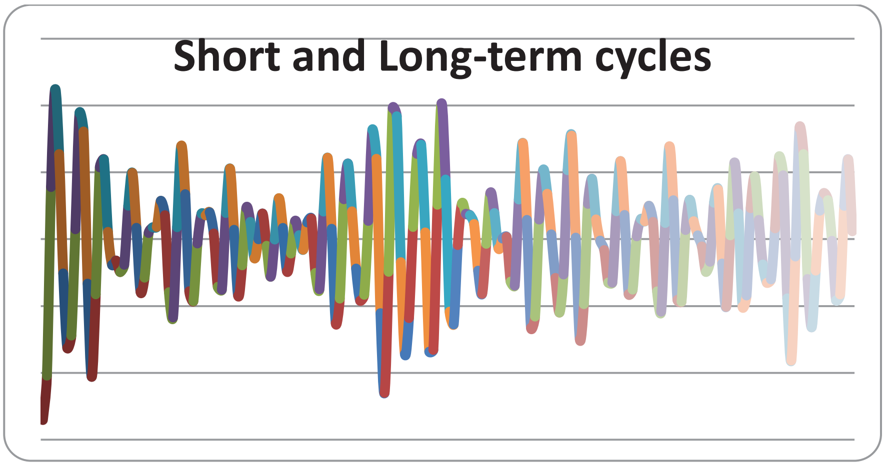
By contrast, in case of finance and industry correlation the difference between the short-term period and the modulated one are significant, the long-term behavior possibly describing the investment cycles associated with industrial development.

On a wider line of thinking the graph of the added cosines (long-term) modulations for all the correlations result in long term cycles with periods of 36 to 41 years. These time constants are similar to the Kondratieff cycles that span on very long periods.



Source: Author's calculations

If we consider the added full cycles (sine and cosine products) then the large cycles have periods of about 24 years. One may assume here that the short term dynamics has an effect of shortening the long term cycles i.e. the present days Kondratieff type cycles may be shorter.



Source: Author's calculations

3.4. Differential equations with external excitation

Based on the pair correlation we pass now to analyze the differential equations of each component considering, this time the evolution is done in a correlated environment described as a sum of sin.cos products of mutual influences as described above. The dynamics of this equivalent potential of correlation is producing a behavior that has more complex pattern.

Further on we will pass to assessing behavior that results from considering this more complex correlation of the specified GDP components. This may result in complex oscillatory behavior to be analyzed below.

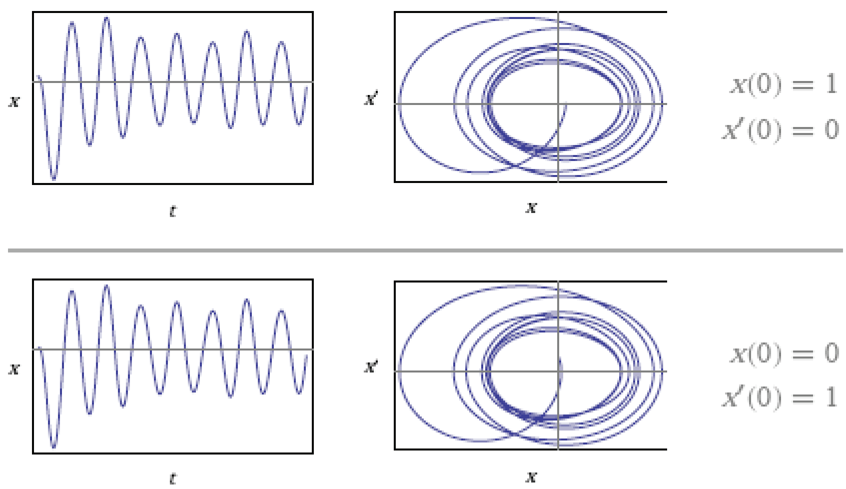
Let us determine the impact into the equations of the GDP components starting with the assumption that the modulated components are introduced in the equations. This is actually changing the character of second order differential equations by added excitation terms showing the interdependence analyzed above. The sum of oscillatory functions sin.cos products (modulated oscillations) resulting from the pairs of mutual influences is added to the equation of each component. This nonlinear term acts as an external excitation of the other components on the one whose behavior is described by the linear second order differential equation determined above. In this way the equations become more close to reality and the resulting behavior of their solutions is analyzed below. For each component the form of the equation will contain a function x(t) that, in each case, refers to the specific component and will have the coefficients and the periods determined above for the case of the natural frequencies (step excitation) and modulated frequencies resulting from combination of the given component with all the others.

To solve these equations we have used Wolfram Alpha website and the results are presented below :

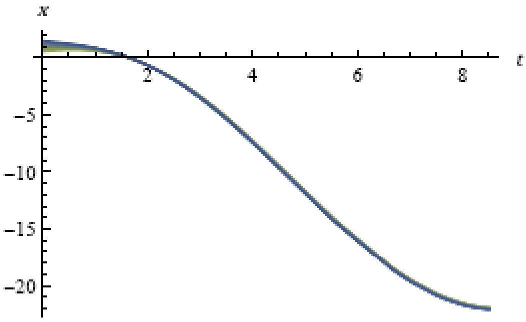
Finance: the equation and solution for this GDP component is given below:

$$1.012\,x''(t) + 0.824\,x'(t) + 0.102\,x(t) + \sin\left(\frac{t}{2.4}\right)\cos\left(\frac{t}{11.8}\right) + \sin\left(\frac{t}{2.89}\right)\cos\left(\frac{t}{69.78}\right) + \sin\left(\frac{t}{2.99}\right)\cos\left(\frac{t}{314}\right) + \sin\left(\frac{t}{3.08}\right)\cos\left(\frac{t}{125}\right) + \sin\left(\frac{t}{2.8}\right)\cos\left(\frac{t}{39.25}\right) = 0$$

Plots of sample individual solutions:



Sample solution family:



(sampling $x(0)$ and $x'(0)$)

Possible Lagrangian:

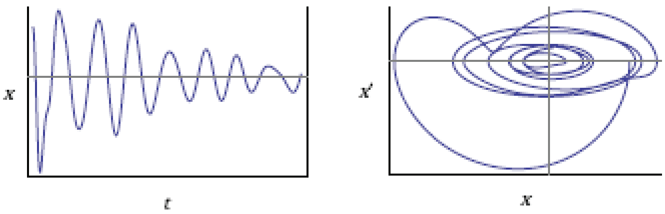
$$\mathcal{L}(x', x, t) =$$

$$\frac{1}{2}\left(-0.100791\, e^{0.814229\, t}\, x^2 + e^{0.814229\, t}\, x'^2 + 2\, e^{0.814229\, t}\, x\,(-0.988142 \sin(0.324675\, t)\cos(0.008\, t) - 0.988142 \sin(0.334448\, t)\cos(0.00318471\, t) - 0.988142 \sin(0.346021\, t)\cos(0.0143308\, t) - 0.988142 \sin(0.357143\, t)\cos(0.0254777\, t) - 0.988142 \sin(0.416667\, t)\cos(0.0847458\, t) + 0.)\right)$$

Industry

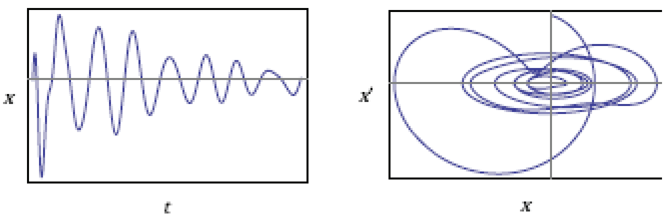
$$2.857\, x''(t) + 1\, x'(t) + 3.656\, x(t) + \sin\left(\frac{t}{2.89}\right)\cos\left(\frac{t}{69.78}\right) + \sin\left(\frac{t}{2.87}\right)\cos\left(\frac{t}{89.7}\right) + \sin\left(\frac{t}{2.33}\right)\cos\left(\frac{t}{14.3}\right) + \sin\left(\frac{t}{2.9}\right)\cos\left(\frac{t}{48.3}\right) + \sin\left(\frac{t}{2.7}\right)\cos\left(\frac{t}{89.7}\right) = 0$$

Plots of sample individual solutions:



$x(0) = 1$

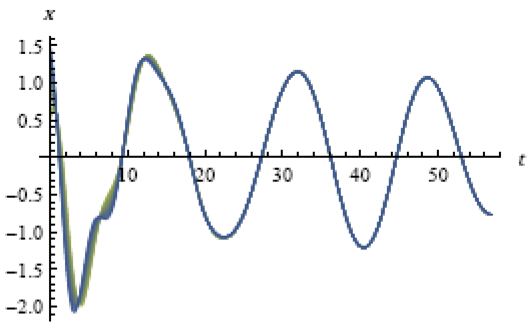
$x'(0) = 0$



$x(0) = 0$

$x'(0) = 1$

Sample solution family:



(sampling $x(0)$ and $x'(0)$)

Possible Lagrangian:

$$\mathcal{L}(x', x, t) =$$
$$\frac{1}{2}\big(-1.27966\, e^{0.350018\, t}\, x^2 + e^{0.350018\, t}\, x'^2 + 2\, e^{0.350018\, t}\, x\,(-0.350018 \sin(0.344828\, t)\cos(0.0207039\, t) -$$
$$0.350018 \sin(0.346021\, t)\cos(0.0143308\, t) - 0.350018 \sin(0.348432\, t)\cos(0.0111483\, t) -$$
$$0.350018 \sin(0.37037\, t)\cos(0.0111483\, t) - 0.350018 \sin(0.429185\, t)\cos(0.0699301\, t) + 0.)\big)$$

3.5. Comments on stability and dynamics of GDP components

The above behavior of the GDP components, when we have considered the coupling among them, shows that except for finance the other components have a convergent dynamic toward some stable cycles. Some of these cycles are more complex while some are simpler but the families of solutions of these equations converge to stable oscillatory behavior. One exception – that was actually underlined earlier – is the behavior of the finance component where the solutions trend is toward increasing negative values i.e. toward instability.

An important thing to notice here is the existence of a Lagrangean for each component. These Lagrangean depend on the component value x, on its derivative x', and also on time t. The time dependence is actually seen at the level of the coefficients.

We may say that out of the three terms in the Lagrange function one is dependent on x^2, the other on x'^2, while the last on x. The literature on the Lagrangean mechanics is very rich and we will only mention a few titles of interest in the references. Let's make now some analogies with the Lagrangean mechanics and see how one may associate the notion of equilibrium for the dynamics of the GDP components, based on Feynman's path integral approach.

Considering the mechanical analogy, the term in x square represents a potential of elastic type – its coefficient could be an elasticity coefficient (not in the sense of the economic notion of 'elasticity'). The term in x'^2 may be associated to a kinetic energy defining the dynamic of each component. The term in x is measuring the influence of the other components having a time dependent coefficient that describes the mutual influence of the other components – this coefficient is a function of sums of sin.cos products depending on short period and long period cycles described above.

In order to analyze the dynamics described by the Lagrangeans we will consider the method introduced by Feynman where in the time dependence of the Lagrangean is eliminated by integrating on given time intervals the Lagrangean and then discussing the behavior described by the resulting Action functional resulting from the integration on each time interval.

If we integrate the Lagrangean (L) on a time interval (since we measure time in Quarters we may consider either one Quarter or 4 Quarters i.e. one year) we get the action associated to the process, in that time interval. Thus, we obtain a functional that has coefficients, which are not time dependent, because they are valid for one Quarter, as we have set the limits of integration.

We may find in each time interval the point of extreme (possibly least) action that is considered to be an equilibrium point for the process in that time interval. These equilibrium points may be different from one time interval to the other and the dynamic evolution of the specific component may be considered to go from one equilibrium, in a given time interval, to the equilibrium in the next time interval.

The evolution trajectory of the process is following the path of least action. The points of least action are the ones given by the annulment of the derivative of the action with x and, they are describing the equilibriums of the process. We stress that in this case we will not have one general equilibrium but, several i.e. one for each time interval of the process.

Thus, the idea of one general equilibrium in economics is only valid in the case the coefficients of the Lagrangean do not depend on time. We have seen in the above considerations that the moment one takes into consideration the reality of the oscillating components of the GDP and their mutual influence, the notion of general equilibrium remains valid only on given time intervals. Moreover, the fact that for each time interval the equilibriums are different, results in different dynamic behavior of the process. The theoretical evidence of this behavior is actually seen in real economic evolution where discontinuous changes occur in combination with smooth behavior and oscillatory periods.

The final analysis we will do here is related to the determination of the Lagrangean for the whole GDP as resulting from the sum of each component's Lagrangean. Then we will integrate the resulting Lagrangean for a series of time intervals to determine the respective action functional followed by a discussion of the type of interval stability and local equilibrium existence.

To determine the sum of Lagrangeans we will express the sin.cos products in each case as sums of sin functions of the determined periods given in table 1 above. Then we will sum all Lagrangean and obtain a possible Lagrangean for the GDP.

The sum of the component's Lagrangeans is presented below:

$$L(x,x',t) = x^2.\left(\frac{1}{2}\right).\sum_{i=1}^{6} c_i\,.e^{b_i t} + x'^2.\left(\frac{1}{2}\right).\sum_{i=1}^{6} e^{b_i t} + x.10.\sum_{i=1}^{6} e^{b_i t}.\sin\left(a_i t\right)$$

The values of the coefficients a_i, b_i, c_i are given in the following table.

Table 4. The values of the coefficients in the summed Lagrangian

i	**component**	c_i	b_i	**T_i (a_i=2p/T_i)**
1	fin	-0.1	0.81	6.02
2	ind	-1.28	0.55	5.55
3	agr	-1.12	0.84	5.93
4	ctr	-2.46	0.71	4
5	com	-0.99	0.51	6.3
6	srv	-1.44	0.56	5.24

Let us determine the formula for the Action (integrated Lagrangean over a time interval) that is the functional we will consider in order to determine the case of least action that may allow considerations on equilibrium in relation with GDP behavior.

Let's first consider that the time dependent coefficients of x' are actually applied to the values of x at the beginning and end of the time interval that are actually defining x', divided by the length of the time interval. If instead of the end values of the interval we consider the average value then we may introduce a variable noted y for each time interval that will actually be connected with x' in the Lagrangean.

Thus the coefficient of x^2 integrated over one quarter, i.e. t=(0,1), is:

$$A = \left(\frac{1}{2}\right).\sum_{i=1}^{6} c_i/b_i\,.e^{b_i t} = -5.21$$

The coefficient of y^2 is:

$$C = \left(\frac{1}{2}\right) \cdot \sum_{i=1}^{6} \frac{1}{b_i} e^{b_i t} = 4.27$$

While the coefficient of x is:

$$B = \frac{1}{a_i^2 + b_i^2} \sum_{i=1}^{6} e^{b_i t} . (b_i sin(a_i t) - a_i cos\,(a_i t) = 15.52$$

The final Action functional $\mathcal{A}$ is of the form:

$$\mathcal{A} = -5.21x^2 + 15.52x + 4.27y^2$$

Let's describe the surface of $\mathcal{A}$ in the phase space (x,y), considering the evolution for each time interval of 1Q from 0 to 16 (i.e. an interval of 4 years).

First the coefficients A, B, and C are given in the next table for the first 8 intervals and the surfaces are presented below.

Table 4. The values for the coefficients of the Action functional for 8 time intervals

Q	A	C	B
1	-5.2	4.3	15.5
2	-10.1	8.4	47.3
3	-19.8	17.0	20.4
4	-39.5	34.7	-119.9
5	-79.6	72.0	-271.1
6	-162.8	151.6	-134.5
7	-337.0	323.3	713.7
8	-706.0	696.9	2456.1

The surfaces of the Action functional are presented below for each time interval. The x values for the extreme values of the surface are in each case either positive or negative. Moreover, the surface is a saddle one that does not have a general equilibrium point but only a maximum for x that is actually a minimum for y. We present the figures for an x-extreme positive and an x-extreme negative value.

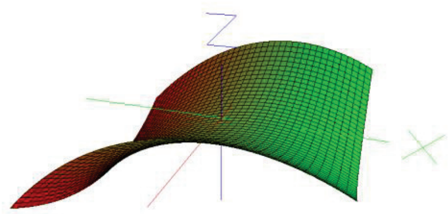


Fig.1.Action functional of GDP for time interval (0,1) – x extreme positive

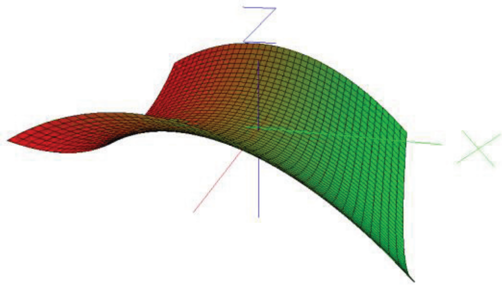


Fig.2. Action functional of GDP for time interval (3,4) – x extreme negative

As we have seen how the Action functional surface is evolving it is clear that the x-extreme values are oscillating from positive to negative and back, this being a representation of the fact that the GDP is not having one general equilibrium but, several dynamical ones. It is also important to note that the surface of the action is a saddle where the extreme point for y is a minimum while the one for x is a maximum.

In order to have a single equilibrium the coefficients of x^2 and of y^2 should have the same sign (negative for an instable equilibrium point and positive for a stable one.

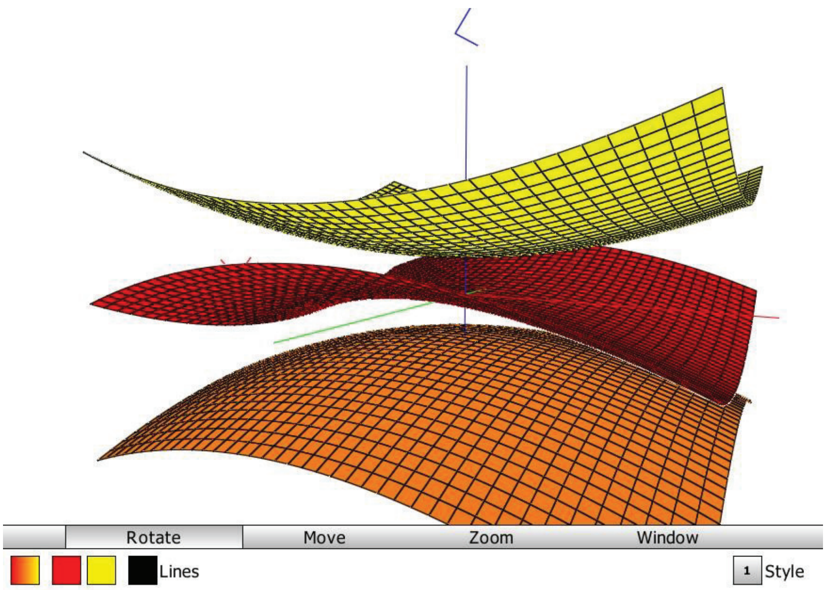


Fig.3. Stable, saddle and unstable equilibrium, Source: Author's calculations

Anyway the situation under consideration is presenting a surface where the behavior may be more complex and crisis could set in more easily. We have only tested the methodology for the set of data from the Romanian economy; it should be logical, though, to launch a wider program for various other economies and various other periods of time, such as to check whether a general stable equilibrium point may show up and what are the conditions for that to happen and to remain constant with time.

4. Conclusions

In the above we have analyzed the oscillatory behavior of the GDP and its components. The data series used for calculations was the one for Romania having a quarterly disaggregation. The components considered were finance, industry, agriculture, commerce, constructions and services. The oscillatory data for each of them was analyzed first by the Fourier transform determining the spectrum of frequencies for each component. Representation of the amplitudes in the complex plane gave important indications on the stable or unstable behavior of each component. In this case finance resulted to have an unstable behavior that may be the prelude of a potential crisis.

Further on we have determined, using an algorithm in MathCad, the coefficients of the second order differential equations associated to these oscillatory data series. The natural frequencies as well as the transient frequencies, for each component, were determined using a step function excitation method. Having determined these frequencies (i.e. the periods of each component's oscillation) we have analyzed the effects of each pair of components. The resulting behavior is similar to the modulated waves in Physics that produce the so called 'beats'. The results of this approach have identified two types of oscillations: (i) one short period process and (ii) a long period one. The long period processes, regrouped into the GDP, are matching the Kondratieff cycles periods while the complete combination of short and long period processes show that the effect of short periods is to shorten the period of the Kondratieff cycles from more than 40 years to les than 30 years for the economic case considered.

All these combinations of oscillations represented under the form of sin.cos products summed for all components, are creating an external excitation potential that is introduced in the differential equations of each component. Solving these equations (by means of Wolfram Alpha website) gives, along with the complex representation of the dynamics in the phase space, the Lagrangean of each component. The sum of the respective Lagrangeans is associated to the GDP total Lagrangean. The analysis of this Lagrangean is aimed at determining the existence of equilibrium points for the behavior of the process. Since the coefficients of the Lagrangean are time dependent and not only dependent on the x and x' (as coordinates associated to the GDP phase space). We have used the Feynman technique of determining the Action functional resulting from the integration of the Lagrangean function on successive time intervals. The existence of equilibrium points is associated to the extreme values of the surface of the Action functional. The representation of the surfaces resulted from the successive time intervals integration of the Lagrangean gives a saddle type of surface having a saddle extreme point (i.e. maximum for x and minimum for x'). The basic results from this approach show that there is not only one equilibrium point (a general equilibrium) for the process but, a dynamic evolution from one time interval equilibrium to the next (as shown by a path integral approach). Moreover, it results that the saddle point behavior is as real as the potential existence of a general equilibrium – be it a stable or an instable one. Further analysis is proposed, on various other economies oscillatory behavior, in order to identify the conditions required for a general equilibrium as well as the requirements for this unique equilibrium point to last.

Finally, we stress that by taking a different view – more nonlinear – to analyze the behavior of the GDP components, leads to the identification of important results related to the existence and types of equilibrium associated to the dynamic of the process. The general equilibrium is but an exception in a process where there are different equilibria for each successive time intervals of the process dynamic evolution. The types of equilibriums encountered may be stable (the usual general equilibrium), unstable, as well as saddle points. The analyzed economy case showed saddle type of successive equilibriums that suggest potential occurrence of unstable behavior. We think this approach is opening the way to more extended research that should cover data series from various types of economies and various periods, in order to identify the conditions for the existence of a general equilibrium and for its persistence.

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