METHODOLOGICAL STUDY REGARDING THE CALCULATION OF STATISTICAL INDICES BY THE PROCESS OF SEPARATING THE ISOLATED ACTION OF EACH FACTOR AND THE PROPORTIONAL DISTRIBUTION OF THE INTERACTION OF THE INFLUENCING FACTORS

Nicolae MIHĂILESCU[1]
"Hyperion" University Bucharest

Carmen UZLĂU[2]
"Hyperion" University Bucharest

Elena Liliana COMAN[3],
IOSUD-SDSE Valahia University of Targoviste, Romania

ABSTRACT: *The methodological study presented in this article offers a solution of practical utility for substantiating decisions aimed at increasing the economic-financial performance of economic operators based on the identification and quantification of the factors that determined the size and modification of an indicator of strong representation of the activity carried out. The methodology presented in this study has a rigorous content, from a mathematical point of view, which respects a principle of calculation and proportional attribution of the influence of each factor that explains the change of a result indicator of the economic activity, synthetic or complex, obtained through sequential contributions but in the same unitary time of two or more factors with different degrees of importance. The general purpose of this methodology is to provide information unaffected by limited, particular principles, with justifications to which more or less pertinent counter-arguments can be brought. It is mentioned that there is the inconvenience of the complexity of the calculations, more difficult to achieve if a manual procedure is used, and a computer solution would be fully recommended.*

Keywords: *statistical index, influence factor, economic indicator*

JEL Classification: *C02*

1. INTRODUCTION

For the substantiation of decisions aimed at the management of economic activity, the **index method** is particularly useful due to the informational content provided by the statistical dimension called the index, obtained as a result of the comparison made in dynamic or static terms.

[1] PhD. Associate Professor, n.mihailescu@yahoo.com
[2] PhD. Associate Professor, carmen_uzlau@yahoo.com
[3] PhD Student, IOSUD-SDSE Valahia University of Targoviste, Romania, lilicoman1978@gmail.com

The statistical index is a relative quantity that expresses one of the following categories of states of economic phenomena:

- *dynamics,*
- *the degree of fulfillment of the programmed or planned indicators,*
- *the relative level of the proposed burden for the increase or decrease of an economic indicator in the following time segment,*
- *the size ratio between two economic indicators identical in terms of content and method of calculation, referring to two similar territorial entities (city, county, country) or two economic agents, but coexisting in time.*

Therefore, the index is the result of the ratio of two statistical indicators referring to the same economic phenomenon, which, in turn, can be presented in absolute, relative or average form. The index expresses the relative change in the size of the indicator from the numerator compared to the size from the denominator of the ratio.

From the point of view of scope, two categories of statistical indices are distinguished: *individual, elementary or simple indices and group indices.*

The individual index expresses the size ratio between two statistical indicators that characterize collections of homogeneous units (objects or types of products) or phenomena with the same economic content. For example, one can calculate the individual index of the dynamics of the physical volume, the dynamics of the prices or the dynamics of the value of the goods sold by an economic agent for each type of goods, separately.

The general formulas for calculating the individual dynamics indices are:

- for a quantitative economic indicator (*f*),

$$i(f) = \frac{f_1}{f_0},$$

- for a qualitative economic indicator (x),

$$i(x) = \frac{x_1}{x_0}$$

- for a complex economic indicator (*Y*),

$$i^{(f)(x)} = \frac{f_1 * x_1}{f_0 * x_0}$$

The group index expresses the average relative change in the characteristics of a collection of units that differ from each other in terms of content or use value.

For example, the group index of the dynamics of the value of goods sold by a commercial company (the group index of the turnover dynamics), which is an index of a complex statistical indicator, is calculated as a ratio between the sum of receipts (value of sales) in the current period or calculation and the amount of receipts (value of sales) from the base period of comparison, according to the following relationship,

$$I^{(q)(p)} = \frac{\sum(q_1 * p_1)}{\sum(q_0 * p_0)}$$

in which,

"q" is the physical volume of sales by type of goods (economic indicator of quantitative type - *f*),

"p" represents the unit sale price for each kind of goods (economic indicator of qualitative type - *x*).

It is stated that both the physical volume of sales and the unit sales prices are not directly summable because they refer to different types of goods and consequently to highlight the separate influence or change of each of the two factors (*q* and *p*) are calculated factorial group indices by applying a certain weighting system.

Therefore, in the case of a factorial-deterministic relationship of the form $Y = f * x$, or $\sum y = \sum(f * x)$ to measure the separate change of each of the factors (quantitative - f and qualitative - x) that determined the change of the complex indicator (Y) or, in another form interpretation method, used only in the case of factorial group indices, to quantify the average change of the quantitative and, respectively, the qualitative indicator, the **method of successive substitutions (in a chain)** is used, as a rule, but, in the analysis practice, other methods are sometimes used weighting methods: Laspeyres Index, Paasche Index, Logarithmic Weights Procedure (Fisher Index), Average Weights Procedure (Edgeworth Index), Finite Increases Procedure (Lagrange Index).

In order to systematize, generalize and rigorously apply the method of successive substitutions, the economic indicators are grouped as follows:

- ***quantitative economic indicators (f),*** such as: physical volume of production or services (q); average number of employees; the time worked by employees, expressed in man-hours; the time worked by employees, expressed in man-days; the average value of fixed assets; the average value of current assets; existing tourist accommodation capacity or the capacity built and intended for tourist accommodation; places-days capacity of existing tourist accommodation; places-days tourist accommodation capacity available, in operation or active; tourists staying in tourist accommodation units; the number of tourist days, etc.;
- ***qualitative economic indicators (x)***: production price or tariff per physical unit of services; retail price; labor costs incurred on average with one employee; full unit cost; the specific consumption of material and energy resources expressed in natural units; expenses per 1000 lei turnover; financial return rate; labor productivity; the average number of rotations of current assets, the average duration in days of a rotation of current assets and in general all indicators expressing economic efficiency;
- ***complex economic indicators (Y)***: turnover; commercial margin; exercise output; added value, total expenses; total revenues; total labor costs; total expenses with raw materials, materials and energy related to the productive activity, trade or service provision; the total consumption of raw materials, materials and fuel expressed in natural units - by types of resources; operating result; the net result of the financial year, the gross result of the year.

The application of the method of successive substitutions implies compliance with the following two basic rules:

1) *the individualization and dimensioning of the influence of a quantitative factor that determined the change of the complex indicator is carried out by weighting (keeping it constant) with the qualitative factor as a basis for comparison*;

2) *the individualization and dimensioning of the influence of a qualitative factor that determined the change in the complex indicator is carried out by weighting with the compared quantitative factor.*

We make it clear that, in the case of the analysis by influence factors of the indicators that characterize the efficiency of the use of direct or primary production factors (labour force, fixed assets and material circulating assets or material and energy resources), consumed to obtain an economic result, the indicators of economic effect are treated as qualitative indicators, and those of economic effort have the meaning and behave as quantitative indicators.

The general calculation formulas used in the case of the method of successive substitutions, when we want to quantify the respective changes in relative and absolute quantities, are the following:

- the total change of the complex phenomenon (indicator):

Index,

$$I^{(f)(x)} = \frac{f_1 * x_1}{f_0 * x_0}, \text{ or } I^{(f)(x)} = \frac{\sum(f_1 * x_1)}{\sum(f_0 * x_0)}$$

The related absolute change,

$$\Delta = f_1 * x_1 - f_0 * x_0, \text{ or } \Delta = \sum(f_1 * x_1) - \sum(f_0 * x_0)$$

from which:

- the influence of the change in the quantitative type factor (*f*):

Index,

$$I(f) = \frac{f_1 * x_0}{f_0 * x_0}, \text{ or } I(f) = \frac{\sum(f_1 * x_0)}{\sum(f_0 * x_0)}$$

The related absolute change,

$$\Delta(f) = f_1 * x_0 - f_0 * x_0, \text{ or } \Delta(f) = \sum(f_1 * x_0) - \sum(f_0 * x_0)$$

- the influence of the change in the qualitative type factor (*x*):

Index,

$$I(x) = \frac{f_1 * x_1}{f_1 * x_0}, \text{ or } I(x) = \frac{\sum(f_1 * x_1)}{\sum(f_1 * x_0)}$$

The related absolute change,

$$\Delta(x) = f_1 * x_1 - f_1 * x_0 \text{ or } \Delta(x) = \sum(f_1 * x_1) - \sum(f_1 * x_0)$$

to check the equalities:

$$I^{(f)(x)} = I(f) * I(x)$$
$$\Delta = \Delta(f) + \Delta(x)$$

2. REFERENCE LITERATURE

Methodological study regarding the calculation of statistical indices through the process of separating the isolated action of each factor and the proportional distribution of the interaction of influencing factors joins the numerous methodological substantiation works that have been presented in articles and specialized papers from the country and abroad.

All the studies we refer to are based on the logic of basing the statistical approach on economic theory. The methods and procedures of statistical processing of statistical information data, the particular cases that aim at the size, structure and dynamics of economic variables, as well as the formation of interdependence relationships between them, are described distinctly.

In this sense, the works that describe the statistical methodology for calculating and interpreting statistical indices, the informational significance of the results, published by Andrei Tudorel, Statistics and econometry, Economic Publishing House, Bucharest, 2003[1], are relevant; Baron T., Biji E., Tövissi L., Wagner P., Isaic-Maniu Al., Korka M., Porojan D., Theoretical and economic statistics, Didactic and Pedagogical Publishing House, Bucharest, 1996[2]; Calot G., Cours de statistique descriptive, DUNOD Publishing House, Paris, 1965[3]; Desabie, J., Theorie et pratique des sondages, Statistique et programs economiques, Volume 10, DUNOD Publishers, Paris, 1966[4]; Isaic-Maniu Al., Mitruţ Constantin, Voineagu Virgil, Statistics for business management, Economic Publishing House, Bucharest, 1995[6];

Mihăilescu, N. - "Statistics and Statistical Bases of Econometrics", Transversal Publishing House, Bucharest, 2021[8]: Mills F. C., Statistical Method, Columbia University Press, New York, 1956[9].

A specialized application treatment refers to the factor analysis of the dynamics of complex indicators which is presented by Mihăilescu, N. in the paper - "Analysis of the economic-financial activity - Research methodologies, solved case studies for the substantiation of economic-financial decisions and knowledge tests" , Transversal Publishing House, Bucharest, 2021[7].

The mentioned works present, in the context of scientifically based methodology, from an economic point of view, statistical studies to express the reality of economic processes with dynamic development or in a static profile.

3. THE METHOD OF SEPARATING THE ISOLATED ACTION OF EACH FACTOR WITH DISTRIBUTION PROPORTIONAL TO THE INTERACTION OF INFLUENCING FACTORS (METHOD OF PROPORTIONAL INCREASES)

Another methodological procedure used to calculate the influence of the factors that determined the change of a complex indicator is known as the ***"Method of separating the isolated action of each factor".***

The application of the principle of separating the individual action of the factors that determine the modification of a complex indicator - presented as a function of two or more influencing factors, according to a factorial-deterministic relationship - is based on a weighting system that invariably uses the basic indicators of comparison, regardless of whether they are of a quantitative or qualitative nature. It results, in this case, and an additional influence which is caused by the interaction of factors or the simultaneous action of factors.

The process of separating the isolated action of each factor leads to the amplification of the volume of calculations, especially when the number of factors that determined the change of the complex indicator is greater than two. To demonstrate this fact, we present the Table 1.

It is noted, thus, the important increase in the typology of factors' interaction indices as the number of indicators considered as influencing factors increases. It is also noted that the influences expressed by the indicators of the interaction of the factors (simultaneous action of the factors) present difficulty for interpretation, and consequently their dimensions are to be distributed over the specified factors, using a criterion of proportionality, thus we have the image of complexity the calculations involved with the application of the procedure of separating the isolated action of each factor.

Note on exemplifying the calculation of the number of combinations,

$$C_3^1 = \frac{3!}{1! * (3-1)!} = \frac{1*2*3}{1*1*2} = 3$$

$$C_5^3 = \frac{5!}{3! * (5-3)!} = \frac{1*2*3*4*5}{1*2*3*1*2} = 10$$

Table 1. The number of statistical indices and related absolute changes in the case of the Procedure of separating the isolated action of each factor

The number of factors	Index synthetic	The number of indices of isolated influences	The number of factor interaction indices						
2 factor	C_2^0	C_2^1	C_2^2						
The number of indices	1	2	1	1					
3 factor	C_3^0	C_3^1	C_3^2	C_3^3					
The number of indices	1	3	3	1	4				
4 factor	C_4^0	C_4^1	C_4^2	C_4^3	C_4^4				
The number of indices	1	4	6	4	1	11			
5 factor	C_5^0	C_5^1	C_5^2	C_5^3	C_5^4	C_5^5			
The number of indices	1	5	10	10	5	1	26		
6 factor	C_6^0	C_6^1	C_6^2	C_6^3	C_6^4	C_6^5	C_6^6		
The number of indices	1	6	15	20	15	6	1	57	
7 factor	C_7^0	C_7^1	C_7^2	C_7^3	C_7^4	C_7^5	C_7^6	C_7^7	
The number of indices	1	7	21	35	35	21	7	1	120

Exemplification of the calculation methodologies will be carried out in the variants that present the complex indicator according to two influencing factors and respectively three influencing factors between which there is a multiplying relationship.

Case 1 – Complex indicator with two influencing factors: “*a*” and “*b*”

- the synthetic index (complex indicator index),

$$I = \frac{a_1 * b_1}{a_0 * b_0} \text{ and}$$

- the absolute change of the complex indicator,

$$\Delta = a_1 * b_1 - a_0 * b_0$$

- indices of the isolated influences of factors “*a*” and “*b*”

a) $$I^{(a)} = \frac{a_1 * b_0}{a_0 * b_0} \text{ and}$$

- the related absolute change,

$$\Delta^{(a)} = (a_1 - a_0) * b_0 = \Delta(a) * b_0$$

b) $$I^{(b)} = \frac{a_0 * b_1}{a_0 * b_0} \text{ and}$$

- the related absolute change,

$$\Delta^{(b)} = (b_1 - b_0) * a_0 = \Delta(b) * a_0$$

- the index of the interaction of factors "*a*" and "*b*"

$$I^{(a)(b)} = \frac{a_1 * b_1}{a_1 * b_0} \div \frac{a_0 * b_1}{a_0 * b_0} = \frac{(a_1 * b_1) * (a_0 * b_0)}{(a_1 * b_0) * (a_0 * b_1)} \text{ and}$$

- the related absolute change,

$$\Delta^{(a)(b)} = a_1 * b_1 - a_1 * b_0 + a_0 * b_0 - a_0 * b_1 = \Delta(a) * \Delta(b) = (a_1 - a_0) * (b_1 - b_0)$$

The recurrence relation between indices (multiplicative format)

$$I = \frac{a_1 * b_1}{a_0 * b_0} = I^{(a)} * I^{(b)} * I^{(a)(b}$$

The recurrence relation between the absolute changes (additive format)

$$\boxed{\Delta = a_1 * b_1 - a_0 * b_0 = \Delta^{(a)} + \Delta^{(b)} + \Delta^{(a)(b)}}$$

After the proportional distribution of the change caused by the interaction of the factors, the factor influences expressed in absolute numbers are:

- the influence of factor "*a*",

$$\Delta^{(a)} = (a_1 - a_0) * b_0 + \frac{(a_1 - a_0) * b_0}{(a_1 - a_0) * b_0 + (b_1 - b_0) * a_0} * \Delta(a) * \Delta(b)$$

- the influence of factor "*b*",

$$\Delta^{(b)} = (b_1 - b_0) * a_0 + \frac{(b_1 - b_0) * a_0}{(a_1 - a_0) * b_0 + (b_1 - b_0) * a_0} * \Delta(a) * \Delta(b)$$

- the proportionality coefficient of the isolated influence, determined by the change in the "*a*" factor,

$$Ka = \frac{(a_1 - a_0) * b_0}{(a_1 - a_0) * b_0 + (b_1 - b_0) * a_0}$$

- the proportionality coefficient of the isolated influence, determined by the change in the "*b*" factor,

$$Kb = \frac{(b_1 - b_0) * a_0}{(a_1 - a_0) * b_0 + (b_1 - b_0) * a_0}$$

Case 2 – Complex indicator with three influencing factors: "*a*", "*b*" and "*c*"

- the synthetic index

$$I = \frac{a_1 * b_1 * c_1}{a_0 * b_0 * c_0} \text{ and}$$

- the absolute change of the complex indicator,

$$\Delta = a_1 * b_1 * c_1 - a_0 * b_0 * c_0$$

- indices of the isolated influences of factors "*a*", "*b*" and "*c*"

$$I^{(a)} = \frac{a_1 * b_0 * c_0}{a_0 * b_0 * c_0} \text{ and}$$

- the related absolute change,

$$\Delta^{(a)} = (a_1 - a_0) * b_0 * c_0 = \Delta(a) * b_0 * c_0$$

b) $I^{(b)} = \frac{a_0 * b_1 * c_0}{a_0 * b_0 * c_0}$ and

- the related absolute change,

$$\Delta^{(b)} = (b_1 - b_0) * a_0 * c_0 = \Delta(b) * a_0 * c_0$$

$I^{(c)} = \frac{a_0 * b_0 * c_1}{a_0 * b_0 * c_0}$ and

- the related absolute change,

$$\Delta^{(c)} = (c_1 - c_0) * a_0 * b_0 = \Delta(c) * a_0 * b_0$$

- indicators of the interaction of factors

1. the interaction of factors "a" and "b"

$$I^{(a)(b)} = \frac{a_1 * b_1 * c_0}{a_1 * b_0 * c_0} \div \frac{a_0 * b_1 * c_0}{a_0 * b_0 * c_0} = \frac{(a_1 * b_1 * c_0) * (a_0 * b_0 * c_0)}{(a_1 * b_0 * c_0) * (a_0 * b_1 * c_0)}$$

and the related absolute change,

$$\Delta^{(a)(b)} = (a_1 * b_1 * c_0) - (a_1 * b_0 * c_0) + (a_0 * b_0 * c_0) - (a_0 * b_1 * c_0)$$
$$= (a_1 - a_0) * (b_1 - b_0) * c_0 = \Delta(a) * \Delta(b) * c_0$$

2. the interaction of factors "a" and "c"

$$I^{(a)(c)} = \frac{a_1 * b_0 * c_1}{a_0 * b_0 * c_1} \div \frac{a_1 * b_0 * c_0}{a_0 * b_0 * c_0} = \frac{(a_1 * b_0 * c_1) * (a_0 * b_0 * c_0)}{(a_0 * b_0 * c_1) * (a_0 * b_0 * c_0)}$$

and the related absolute change,

$$\Delta^{(a)(c)} = (a_1 * b_0 * c_1) - (a_0 * b_0 * c_1) + (a_0 * b_0 * c_0) - (a_1 * b_0 * c_0) =$$
$$= (a_1 - a_0) * (c_1 - c_0) * b_0 = \Delta(a) * \Delta(c) * b_0$$

3. the interaction of factors "b" şi "c"

$$I^{(b)(c)} = \frac{a_0 * b_1 * c_1}{a_0 * b_1 * c_0} \div \frac{a_0 * b_0 * c_1}{a_0 * b_0 * c_0} = \frac{(a_0 * b_1 * c_1) * (a_0 * b_0 * c_0)}{(a_0 * b_1 * c_0) * (a_0 * b_0 * c_1)}$$

and the related absolute change,

$$\Delta^{(b)(c)} = (a_0 * b_1 * c_1) - (a_0 * b_1 * c_0) + (a_0 * b_0 * c_0) - (a_0 * b_0 * c_1) =$$
$$= (b_1 - b_0) * (c_1 - c_0) * a_0 = \Delta(b) * \Delta(c) * a_0$$

4. the interaction of factors "a" , "b" and "c"

$$I^{(a)(b)(c)} = \left[\frac{a_1 * b_1 * c_1}{a_0 * b_1 * c_1} \div \frac{a_1 * b_1 * c_0}{a_0 * b_1 * c_0}\right] \div \left[\frac{a_1 * b_0 * c_1}{a_0 * b_0 * c_1} \div \frac{a_1 * b_0 * c_0}{a_0 * b_0 * c_0}\right] =$$

$$= \frac{(a_1 * b_1 * c_1) * (a_0 * b_1 * c_0) * (a_0 * b_0 * c_1) * (a_1 * b_0 * c_0)}{(a_0 * b_1 * c_1) * (a_1 * b_1 * c_0) * (a_1 * b_0 * c_1) * (a_0 * b_0 * c_0)}$$

and the related absolute change,

$$\Delta^{(a)(b)(c)} = (a_1 * b_1 * c_1) + (a_0 * b_1 * c_0) + (a_0 * b_0 * c_1) + (a_1 * b_0 * c_0) - (a_0 * b_1 * c_1)$$
$$-$$

$$-(a_1 * b_1 * c_0) - (a_1 * b_0 * c_1) - (a_0 * b_0 * c_0) = (a_1 - a_0) * (b_1 - b_0) * (c_1 - c_0) = \Delta(a) * \Delta(b) * \Delta(c)$$

The recurrence relation between indices (multiplicative format),

$$I = \frac{a_1 * b_1 * c_1}{a_0 * b_0 * c_0} = I^{(a)} * I^{(b)} * I^{(c)} * I^{(a)(b)} * I^{(a)(c)} * I^{(b)(c)} * I^{(a)(b)(c)}$$

The recurrence relation between the absolute changes (additive format),

$$\Delta = a_1 * b_1 * c_1 - a_0 * b_0 * c_0 = \Delta^{(a)} + \Delta^{(b)} + \Delta^{(c)} + \Delta^{(a)(b)} + \Delta^{(a)(c)} + \Delta^{(b)(c)} + \Delta^{(a)(b)(c)}$$

After the proportional distribution of the changes caused by the interaction of the factors with the changes calculated by the isolated substitution of each factor, the factor influences expressed in absolute numbers (derivative procedure in additive format) are:

- the influence of factor "a",

$$\Delta^{(a)} = \Delta(a) * b_0 * c_0 + \frac{\Delta(a) * b_0 * c_0}{\Delta(a) * b_0 * c_0 + \Delta(b) * a_0 * c_0 + \Delta(c) * a_0 * b_0} * \Delta(a) * \Delta(b) * \Delta(c) + \\ + \frac{\Delta(a) * b_0 * c_0}{\Delta(a) * b_0 * c_0 + \Delta(b) * a_0 * c_0} * \Delta(a) * \Delta(b) * c_0 + \frac{\Delta(a) * b_0 * c_0}{\Delta(a) * b_0 * c_0 + \Delta(c) * a_0 * b_0} * \Delta(a) * \Delta(c) * b_0$$

- the influence of factor "b",

$$\Delta^{(b)} = \Delta(b) * a_0 * c_0 + \frac{\Delta(b) * a_0 * c_0}{\Delta(a) * b_0 * c_0 + \Delta(b) * a_0 * c_0 + \Delta(c) * a_0 * b_0} * \Delta(a) * \Delta(b) * \Delta(c) + \\ + \frac{\Delta(b) * a_0 * c_0}{\Delta(a) * b_0 * c_0 + \Delta(b) * a_0 * c_0} * \Delta(a) * \Delta(b) * c_0 + \frac{\Delta(b) * a_0 * c_0}{\Delta(b) * a_0 * c_0 + \Delta(c) * a_0 * b_0} * \Delta(b) * \Delta(c) * a_0$$

- the influence of factor "c",

$$\Delta^{(c)} = \Delta(c) * a_0 * b_0 + \frac{\Delta(c) * a_0 * b_0}{\Delta(a) * b_0 * c_0 + \Delta(b) * a_0 * c_0 + \Delta(c) * a_0 * b_0} * \Delta(a) * \Delta(b) * \Delta(c) + \\ + \frac{\Delta(c) * a_0 * b_0}{\Delta(a) * b_0 * c_0 + \Delta(c) * a_0 * b_0} * \Delta(a) * \Delta(c) * b_0 + \frac{\Delta(c) * a_0 * b_0}{\Delta(b) * a_0 * c_0 + \Delta(c) * a_0 * b_0} * \Delta(b) * \Delta(c) * a_0$$

The exposed methodology has a rigorous content that respects a principle of calculation and proportional attribution of the influence factors that explain the change of an economic activity result indicator, synthetic or complex, obtained through sequential but at the same time unitary contributions of two or more factors with different degrees of importance. The general purpose of this methodology is to provide information unaffected by limited, particular principles, with justifications to which more or less pertinent counter-arguments can be brought.

It is mentioned that there is the inconvenience of the complexity of the calculations, more difficult to achieve if a manual procedure is used, and a computer solution would be fully recommended.

- **Case study, demonstrative**

It is mentioned that in order to make a convenient demonstration, the derivative procedure will be applied in additive format and the statistical data that will be used are conventional. The complex indicator ("Y") under analysis is presented as a product of a number of three influence factor indicators, ”*a*”, ”*b*” and ”*c*”.

Table 2 The system of statistical data on the dynamics of the "Y" indicator and influencing factors "a", "b" and "c".

The name of the indicators		Base period (Measurement units – m.u.)	Calculation period (Measurement units –m.u.)	Dynamics indices
The complex indicator		$Y_0 = a_0 * b_0 * c_0 = 90$	$Y_1 = a_1 * b_1 * c_1 = 192$	2,13333
Factorial indicators	*a*	3	4	1,33333
	b	5	6	1,20000
	c	6	8	1,33333

The total absolute change of the complex indicator, in the calculation period compared to the base period, is given by the following relationship:

$$\Delta = a_1 * b_1 * c_1 - a_0 * b_0 * c_0 = 192 - 90 = 102,$$ from which :

- the isolated influence of the indicator (factor) "a"

$$\Delta^{(a)} = (a_1 - a_0) * b_0 * c_0 = (4-3) * 5 * 6 = +30$$

- the isolated influence of the indicator (factor) "b"

$$\Delta^{(b)} = (b_1 - b_0) * a_0 * c_0 = (6-5) * 3 * 6 = +18$$

- the isolated influence of the indicator (factor) „c”

$$\Delta^{(c)} = (c_1 - c_0) * a_0 * b_0 = (8-6) * 3 * 5 = +30$$

- simultaneous influence of indicators (interaction of factors) „a” and „b”

$$\Delta^{(a)(b)} = (a_1 - a_0) * (b_1 - b_0) * c_0 = (4-3) * (6-5) * 6 = +6$$

- simultaneous influence of indicators (interaction of factors) „a” and „c”

$$\Delta^{(a)(c)} = (a_1 - a_0) * (c_1 - c_0) * b_0 = (4-3) * (8-6) * 5 = +10$$

- simultaneous influence of indicators (interaction of factors) „b” and „c”

$$\Delta^{(b)(c)} = (b_1 - b_0) * (c_1 - c_0) * a_0 = (6-5) * (8-6) * 3 = +6$$

- simultaneous influence of indicators (interaction of factors) „a”, „b” and „c”

$$\Delta^{(a)(b)(c)} = (a_1 - a_0) * (b_1 - b_0) * (c_1 - c_0) = (4-3) * (6-5) * (8-6) = +2$$

After applying the procedure for distributing the interjection of the factors, the result is:

- the influence of the factor „a”,

$$\Delta^{(a)} = \Delta(a) * b_0 * c_0 + \frac{\Delta(a) * b_0 * c_0}{\Delta(a) * b_0 * c_0 + \Delta(b) * a_0 * c_0 + \Delta(c) * a_0 * b_0} *$$
$$* \Delta(a) * \Delta(b) * \Delta(c) + \frac{\Delta(a) * b_0 * c_0}{\Delta(a) * b_0 * c_0 + \Delta(b) * a_0 * c_0} * \Delta(a) * \Delta(b) * c_0 +$$

$$+\frac{\Delta(a)*b_0*c_0}{\Delta(a)*b_0*c_0+\Delta(c)*a_0*b_0}*\Delta(a)*\Delta(c)*b_0=$$
$$=30+\frac{30}{30+18+30}*2+\frac{30}{30+18}*6+\frac{30}{30+30}*10=$$

=30.00000+0.76923+3.75000+5.00000=+39.51923

- the influence of the factor „*b*”,

$$\Delta^{(b)}=\Delta(b)*a_0*c_0+\frac{\Delta(b)*a_0*c_0}{\Delta(a)*b_0*c_0+\Delta(b)*a_0*c_0+\Delta(c)*a_0*b_0}*$$
$$*\Delta(a)*\Delta(b)*\Delta(c)+\frac{\Delta(b)*a_0*c_0}{\Delta(a)*b_0*c_0+\Delta(b)*a_0*c_0}*\Delta(a)*\Delta(b)*c_0+$$
$$+\frac{\Delta(b)*a_0*c_0}{\Delta(b)*a_0*c_0+\Delta(c)*a_0*b_0}*\Delta(b)*\Delta(c)*a_0=$$
$$=18+\frac{18}{30+18+30}*2+\frac{18}{30+18}*6+\frac{18}{18+30}*6=$$

=18.00000+0.46154+2.25000+2.25000=+22.96154

- the influence of the factor „*c*”,

$$\Delta^{(c)}=\Delta(c)*a_0*b_0+\frac{\Delta(c)*a_0*b_0}{\Delta(a)*b_0*c_0+\Delta(b)*a_0*c_0+\Delta(c)*a_0*b_0}*$$
$$*\Delta(a)*\Delta(b)*\Delta(c)+\frac{\Delta(c)*a_0*b_0}{\Delta(a)*b_0*c_0+\Delta(c)*a_0*b_0}*\Delta(a)*\Delta(c)*b_0+$$
$$+\frac{\Delta(c)*a_0*b_0}{\Delta(b)*a_0*c_0+\Delta(c)*a_0*b_0}*\Delta(b)*\Delta(c)*a_0=$$
$$=30+\frac{30}{30+18+30}*2+\frac{18}{30+30}*10+\frac{30}{18+30}*6=$$

=30.00000+0.76923+5.00000+3.75000=+39.51923

To calculate the factorial influences that determined the change in the complex indicator, the following proportionality coefficients were used:

1) for the distribution of the interaction of the factors *”a*”, *”b*” şi *”c*”

- the proportionality coefficient of the isolated influence, determined by the change of the factor *”a*”,

$$K(a)bc=\frac{\Delta(a)*b_0*c_0}{\Delta(a)*b_0*c_0+\Delta(b)*a_0*c_0+\Delta(c)*a_0*b_0}=\frac{30}{30+18+30}=0{,}384615$$

- the proportionality coefficient of the isolated influence, determined by the change of the factor *”b*”,

$$Ka(b)c=\frac{\Delta(b)*a_0*c_0}{\Delta(a)*b_0*c_0+\Delta(b)*a_0*c_0+\Delta(c)*a_0*b_0}=\frac{18}{30+18+30}=0{,}230769$$

- the proportionality coefficient of the isolated influence, determined by the change of the factor *”c*”,

$$Kab(c)=\frac{\Delta(c)*a_0*b_0}{\Delta(a)*b_0*c_0+\Delta(b)*a_0*c_0+\Delta(c)*a_0*b_0}=\frac{30}{30+18+30}=0{,}384615$$

2) for the distribution of the interaction of the factors *”a*” and *”b*”

- the proportionality coefficient of the isolated influence, determined by the change of the factor *"a"*,

$$K(a)b = \frac{\Delta(a)*b_0*c_0}{\Delta(a)*b_0*c_0+\Delta(b)*a_0*c_0} = \frac{30}{30+18} = 0{,}625$$

- the proportionality coefficient of the isolated influence, determined by the change of the factor *"b"*,

$$Ka(b) = \frac{\Delta(b)*a_0*c_0}{\Delta(a)*b_0*c_0+\Delta(b)*a_0*c_0} = \frac{18}{30+18} = 0{,}375$$

3) for the distribution of the interaction of the factors *"a"* and *"c"*

- the proportionality coefficient of the isolated influence, determined by the change of the factor *"a"*,

$$K(a)c = \frac{\Delta(a)*b_0*c_0}{\Delta(a)*b_0*c_0+\Delta(c)*a_0*b_0} = \frac{30}{30+30} = 0{,}500$$

- the proportionality coefficient of the isolated influence, determined by the change of the factor *"c"*,

$$Ka(c) = \frac{\Delta(c)*a_0*b_0}{\Delta(a)*b_0*c_0+\Delta(c)*a_0*b_0} = \frac{30}{30+30} = 0{,}500$$

4) for the distribution of the interaction of the factors *"b"* and *"c"*

- the proportionality coefficient of the isolated influence, determined by the change of the factor *"b"*,

$$K(b)c = \frac{\Delta(b)*a_0*c_0}{\Delta(b)*a_0*c_0+\Delta(c)*a_0*b_0} = \frac{18}{18+30} = 0{,}375$$

- the proportionality coefficient of the isolated influence, determined by the change of the factor *"c"*,

$$Kb(c) = \frac{\Delta(c)*a_0*b_0}{\Delta(b)*a_0*c_0+\Delta(c)*a_0*b_0} = \frac{30}{18+30} = 0{,}625$$

The total absolute change of the complex indicator: +102.00000 monetary units.
from which:

- the influence of the factor *"a"*: +39.51923 monetary units.
- the influence of the factor *"b"*: +22.96154 monetary units.
- the influence of the factor *"c"*: +39.51923 monetary units.

Based on these results, the following findings are identified:

- the complex indicator registered an increase in the calculation period compared to the level of the base period by 2.1333 times, respectively by 102,000 m.u;
- factor "*a*" caused the increase of the complex indicator "*Y*" by 39.51923 m.u. respectively by 38.744%;
- factor "*b*" justifies the increase of the complex indicator "*Y*" by 22.96154 m.u. respectively by 22.511%;
- factor "*c*" caused the increase of the complex indicator "*Y*" by 39.51923 m.u. respectively by 38.744%;

Note: A customized situation for a complex indicator like "*Turnover*", the following functional relation, *Y =f(a,b,c)*, can be written:

*Turnover (Y) = Stock turnover rate expressed in number of turnovers (a) * Proportion of stocks in the value of current assets (b) * Value of current assets (c)*

It is specified that the value of stocks and respectively the value of current assets are calculated as average values related to a period of time for which the turnover was recorded.

Case 3 – Complex indicator with four influencing factors: "a", "b", "c" and "d",
Example of a demonstrative calculation with conventional statistical data where the complex indicator Y is in a determining relationship with four factorial indicators (Method of proportional increases – additive variant)

Base period: $Y_0 = a_0 \cdot b_0 \cdot c_0 \cdot d_0 = 3 \cdot 5 \cdot 7 \cdot 9 = 945$ m.u.
Calculation period: $Y_1 = a_1 \cdot b_1 \cdot c_1 \cdot d_1 = 4 \cdot 6 \cdot 8 \cdot 10 = 1920$ m.u.

The total absolute change of the *Y* indicator,
$\Delta = Y_1 - Y_0 = 1920 - 945 = 975$ m.u.

The isolated influence of each factor:
The isolated influence of the factor "*a*",
$\Delta(a{:}\,bcd) = (a_1 - a_0) \cdot b_0 \cdot c_0 \cdot d_0 = (4 - 3) \cdot 5 \cdot 7 \cdot 9 = 315$
The isolated influence of the factor "*b*",
$\Delta(b{:}\,acd) = (b_1 - b_0) \cdot a_0 \cdot c_0 \cdot d_0 = (6 - 5) \cdot 3 \cdot 7 \cdot 9 = 189$
The isolated influence of the factor "*c*",
$\Delta(c{:}\,abd) = (c_1 - c_0) \cdot a_0 \cdot b_0 \cdot d_0 = (8 - 7) \cdot 3 \cdot 5 \cdot 9 = 135$
The isolated influence of the factor "*d*",
$\Delta(d{:}\,abc) = (d_1 - d_0) \cdot a_0 \cdot b_0 \cdot c_0 = (10 - 9) \cdot 3 \cdot 5 \cdot 7 = 105$
TOTAL: 315 + 189 + 135 + 105 = 744

Calculation of the proportion of the isolated influence of each factor in the total change of the isolated influences:
Proportion of the factor's isolated influence "*a*" : 315/744 = 0,423387
Proportion of the factor's isolated influence "*b*" : 189/744 = 0,254032
Proportion of the factor's isolated influence "*c*" : 135/744 = 0,181452
Proportion of the factor's isolated influence "*d*" : 105/744 = 0,141129
TOTAL: 0,423387 + 0,254032 + 0,181452 + 0,141129 = 1,000000

- **The influence of the interaction of factors:**
- Group of 2 factors
The influence of the interaction of factors "*a*" and "*b*"
$\Delta(a, b) = (a_1 - a_0) \cdot (b_1 - b_0) \cdot c_0 \cdot d_0 = 1 \cdot 1 \cdot 7 \cdot 9 = 63$
The influence of the interaction of factors "*a*" and "*c*"
$\Delta(a, c) = (a_1 - a_0) \cdot (c_1 - c_0) \cdot b_0 \cdot d_0 = 1 \cdot 1 \cdot 5 \cdot 9 = 45$
The influence of the interaction of factors "*a*" and "*d*"
$\Delta(a, d) = (a_1 - a_0) \cdot (d_1 - d_0) \cdot b_0 \cdot c_0 = 1 \cdot 1 \cdot 5 \cdot 7 = 35$
The influence of the interaction of factors "*b*" and "*c*"
$\Delta(b, c) = (b_1 - b_0) \cdot (c_1 - c_0) \cdot a_0 \cdot d_0 = 1 \cdot 1 \cdot 3 \cdot 9 = 27$
The influence of the interaction of factors "*b*" and "*d*"
$\Delta(b, d) = (b_1 - b_0) \cdot (d_1 - d_0) \cdot a_0 \cdot c_0 = 1 \cdot 1 \cdot 3 \cdot 7 = 21$
The influence of the interaction of factors "*c*" and "*d*"
$\Delta(c, d) = (c_1 - c_0) \cdot (d_1 - d_0) \cdot a_0 \cdot b_0 = 1 \cdot 1 \cdot 3 \cdot 5 = 15$
TOTAL: 63 + 45 + 35 + 27 + 21 + 15 = 206

SELECTION OF INTERACTIONS RELATED TO 2 FACTORS:
Factor interactions "a" = 63 + 45 + 35 = 143
Factor interactions "b" = 63 + 21 + 27 = 111
Factor interactions "c" = 45 + 27 + 15 = 87
Factor interactions "d" = 21 + 15 + 35 =71
Total = 143 + 111 + 67 + 71 = 412

PROPORTIONS of the interaction in the case of the group of 2 factors:
Proportion of factor interactions "a" = 143/412 = 0,3470874
Proportion of factor interactions "b" = 111/412 = 0,2694174
Proportion of factor interactions "c" = 87/412 = 0,2111651
Proportion of factor interactions "d" = 71/412 = 0,1723301

PROPORTIONAL DISTRIBUTION OF THE INTERACTIONS OF 2 FACTORS ON THE INFLUENCE OF EACH FACTOR
For the factor "a" = 206 x 0,3470874 = 71,5
For the factor "b" = 206 x 0,2694174 = 55,5
For the factor "c" = 206 x 0,2111651 = 43,5
For the factor "d" = 206 x 0,1723301 = 35,5
Total = 71,5 + 55,5 + 43,5 + 35,5 = 206,0

Group of 3 factors
The influence of the interaction of factors "a", "b" and "c"
$\Delta(a, b, c) = (a_1 - a_0) \cdot (b_1 - b_0) \cdot (c_1 - c_0) \cdot d_0 = 1 \cdot 1 \cdot 1 \cdot 9 = 9$
The influence of the interaction of factors "a" , "b" and "d"
$\Delta(a, b, d) = (a_1 - a_0) \cdot (b_1 - b_0) \cdot (d_1 - d_0) \cdot c_0 = 1 \cdot 1 \cdot 1 \cdot 7 = 7$
The influence of the interaction of factors "a" , "c" and "d"
$\Delta(a, c, d) = (a_1 - a_0) \cdot (c_1 - c_0) \cdot (d_1 - d_0) \cdot b_0 = 1 \cdot 1 \cdot 1 \cdot 5 = 5$
The influence of the interaction of factors "b" , "c" and "d"
$\Delta(b, c, d) = (b_1 - b_0) \cdot (c_1 - c_0) \cdot (d_1 - d_0) \cdot a_0 = 1 \cdot 1 \cdot 1 \cdot 3 = 3$
TOTAL: 9 + 7 + 5 + 3 =24

SELECTION OF INTERACTIONS RELATED TO 3 FACTORS:
Factor interactions: "a" = 9 + 7 + 5 = 21
Factor interactions: "b" = 9 + 7 + 3 = 19
Factor interactions: "c" = 9 + 5 + 3 = 17
Factor interactions: "d" = 7 + 5 + 3 =15
Total = 21 + 19 + 17 + 15 = 72

PROPORTIONS of the interaction in the case of the group of 3 factors:
Proportion of factor interactions "a" = 21/72 = 0,2916667
Proportion of factor interactions "b" = 19/72 = 0,2638889
Proportion of factor interactions "c" = 17/72 = 0,2361111
Proportion of factor interactions "d" = 16/72 = 0,2083333

PROPORTIONAL DISTRIBUTION OF THE INTERACTIONS OF 3 FACTORS ON THE INFLUENCE OF EACH FACTOR
For the factor "a" = 24 x 0,2916667 = 7,0
For the factor "b" = 24 x 0,2638889 = 6,3
For the factor "c" = 24 x 0,2361111 = 5,7

For the factor "*d*" = 24 x 0,2083333 = 5,0
Total = 7,0 + 6,3 + 5,7 + 5,0 = 24,0

Group of 4 factors
The influence of the interaction of factors "*a*" ,"*b*", "*c*" and "*d*"
$\Delta(a, b, c, d) = (a_1 - a_0) \cdot (b_1 - b_0) \cdot (c_1 - c_0) \cdot (d_1 - d_0) = 1 \cdot 1 \cdot 1 \cdot 1 = 1$

PROPORTIONAL DISTRIBUTION OF THE INTERACTION OF THE 4 FACTORS ON THE INFLUENCE OF EACH FACTOR
For the factor "*a*" = 1 x 0,423387 = 0,423387
For the factor "*b*" = 1 x 0,254032 = 0,254032
For the factor "*c*" = 1 x 0,181452 = 0,181452
For the factor "*d*" = 1 x 0,141129 = 0,141129
Total = 0,423387 + 0,254032 + 0,181452 + 0,141129 **= 1,0**

The relationship to confirm the correctness of the calculations,
$\Delta = Y_1 - Y_0$**= 1920 – 945 = 975 = 744 +206 + 24 + 1**

The explicit influences of each factor that determined the change in the complex indicator Y, calculations:
Influence of the factor "*a*",
$\Delta^{(a)}$ = 315,00000 + 71,50000 + 7,00000 + 0,423387 = 393, 923387 m.u.
Influence of the factor "*b*",
$\Delta^{(b)}$ = 189,00000 + 55,50000 + 6,30000 + 0,254032 = 251,054032 m.u.
Influence of the factor "*c*",
$\Delta^{(c)}$ 135,00000 + 43,50000 + 5,70000 + 0,181452 = 184,381452 m.u..
Influence of the factor "*d*",
$\Delta^{(d)}$ = 105,00000 + 35,50000 + 5,00000 + 0,141129 = 145,641129 m.u.
TOTAL:393, 923387 + 251,054032 + 184,381452 + 145,641129 **= 975,00000** m.u.

Note. A particular situation for a complex indicator such as "Turnover" can be considered the following functional relationship:

Turnover (Y) = Average number of staff (a) x Degree of technical endowment of work with fixed assets that have an active role in the economic process (b) x Production of the year that returns to 1 leu fixed assets with an active role in the economic process (c) x Degree of capitalization of the production of exercise (d)

It is specified that the value of fixed assets that have an active role in the economic process is included in the model as an average value related to the time period comparable to that to which the turnover refers.

This methodology, exemplified for four influencing factors, is also applicable to complex indicators that are explained by a relationship with 5 or more factors.

REFERENCES

1. Andrei Tudorel, *Statistică şi econometrie*, Editura Economică, Bucureşti, 2003
2. Baron T., Biji E., Tövissi L., Wagner P., Isaic-Maniu Al., Korka M., Porojan D., *Statistică teoretică şi economică*, Editura Didactică şi Pedagogică, Bucureşti, 1996
3. Calot G., *Cours de statistique descriptiv*e, Editura DUNOD, Paris, 1965
4. Desabie, J., *Theorie et pratique des sondages, Statistique et programmes economiques*, Volumul 10, Editura DUNOD, Paris, 1966

5. Dobrescu, Emilian, *Ritmul creşterii economice*, Editura politică, Bucureşti, 1968
6. Isaic-Maniu Al., Mitruţ Constantin, Voineagu Virgil, *Statistica pentru managementul afacerilor,* Editura Economică, Bucureşti, 1995
7. Mihăilescu Nicolae, *Analiza activităţii economico-financiare – Metodologii de cercetare, studii de caz rezolvate pentru fundamentarea deciziilor economico – financiare şi teste de cunoştinţe*", Editura Transversal, București, 2021
8. Mihăilescu Nicolae, *Statistică și Bazele statistice ale econometriei*, Editura Transversal, București, 2021
9. Mills F. C., *Statistical Method*, Columbia University Press, New York, 1956